Beam Fit Algorithms

Introduction

There are no formal standards that define how beam profile fits should be done. Some aspects of our approach are generally accepted industry practice. Some represent specific customer requests that have been incorporated into the software. This application note describes the following algorithms:

- **Gaussian Fit**
  - **GFit**
  - **G 2W**
  - **Fit Coefficient**
  - **Roughness**
  - **Max Deviation**
  - **Standard Deviation**
- **Top Hat Fit**
  - **Max Deviation**
  - **Standard Deviation**
- **Non-uniformity**

These options are accessed by right-clicking on the graph-like **profile** areas in the lower part of the main screen (Fig. 1). In all of the following calculations, the software first determines and subtracts the baseline.

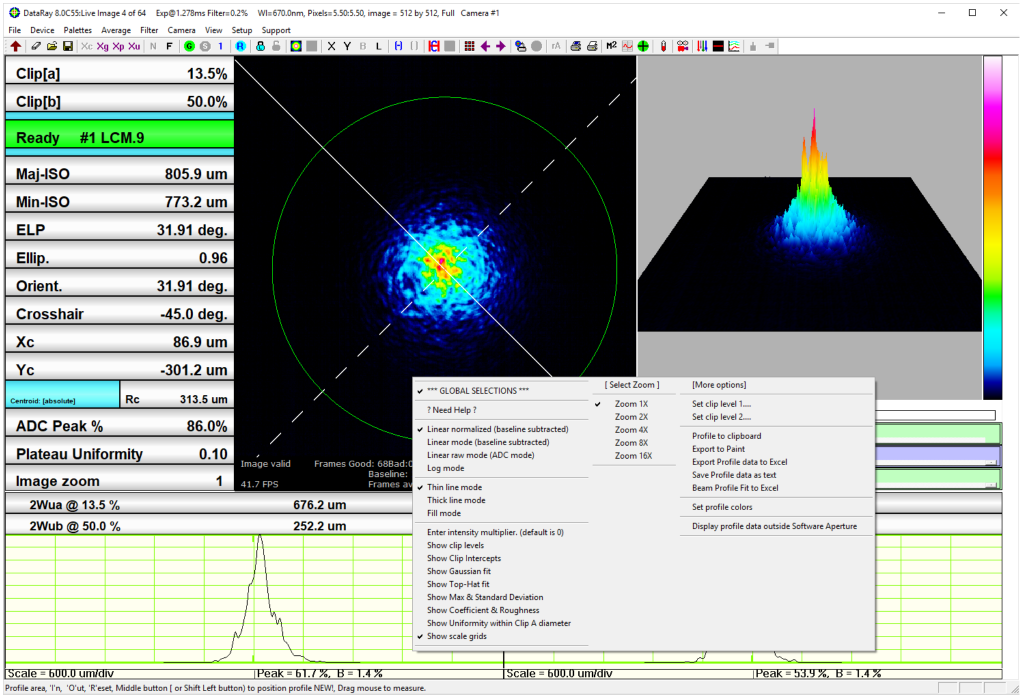

Figure 1: The menu that appears upon right-click of the profile area. The algorithms described in this paper are toggled by clicking the appropriate option in this menu.

Gaussian Fit

GFit and G 2W

When **Show Gaussian fit** is enabled, **GFit** and **G 2W** results buttons appear above the profile area, and a Gaussian curve, drawn in red, is superimposed over the profile (Fig. 2) The Gaussian fit is based upon a fit algorithm that, while keeping the centroid the same as that of the profile, iteratively adjusts the height and width of the Gaussian curve until the least squares difference between the actual profile and the Gaussian profile is minimized. Specifically, the steps are:

1. Set the centroid position of the fitted Gaussian to the same as that of the actual profile.
2. The best fit iteration starts from the actual 4-sigma diameter.
3. The initial height of the Gaussian fit is set to the peak intensity of the actual profile.
4. Iteratively adjust the height and width of the Gaussian fit until the least squares difference is minimized.
5. Let $\mathbf{Sum_{diff}}$ represent the absolute differences between each profile data point and its corresponding Gaussian fit data point.
6. Let **A** represent the total area under the Gaussian fit curve.
7. **GFit** is calculated as a percentage using:

$$GFit = 100 * \frac{A - Sum_{diff}}{A}$$

8. **G 2W** is the diameter of the least squares fitted Gaussian at 13.5% clip.

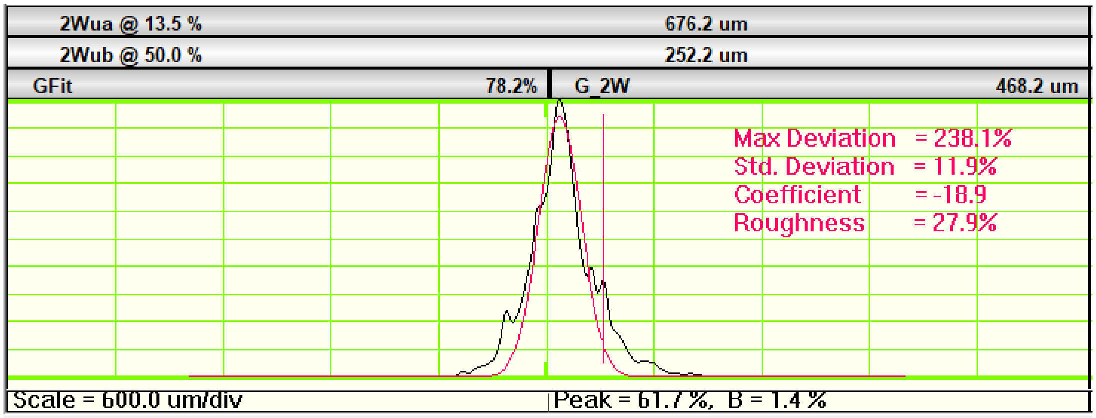

Figure 2: When **Show Gaussian fit** is enabled, the **GFit** and **G_2W** results buttons are displayed above the profile area. The Gaussian fit curve is superimposed in the color red. The vertical red line marks the point of **Max Deviation** between the Gaussian fit curve and the profile data.

Fit Coefficient and Roughness

The **Gaussian Fit Coefficient** and **Gaussian Roughness** are calculated as follows:

1. The variable **N** represents the total number of points in the profile. Find the average between the actual point, $\mathbf{P_j}$, and the fitted Gaussian point, $\mathbf{G_j}$, for all points:

$$A = \frac{\sum_{j=0}^{N}(P_j - G_j)}{N}$$

2. For each point, determine the difference, $\mathbf{D_j}$, from the average of the deviation:

$$D_j = (P_j - G_j) - A$$

3. Determine the sum of $\mathbf{D_j^2}$ and call this sum **S**:

$$S = \sum (D_j^2)$$

4. Determine the **Gaussian Fit Coefficient, C**:

$$C = 1 - \frac{\sqrt{\frac{S}{N}}}{N}$$

5. Define $\mathbf{Max_{Pj}}$ as the maximum $\mathbf{P_j}$ of all points in the profile data.

6. Define $\mathbf{Max_{(Pj - Gj)}}$ as the maximum ($\mathbf{P_j}$ **-** $\mathbf{G_j}$) of all points in the profile data.

7. Determine the **Gaussian Roughness Coefficient, R**:

$$R = 100 * \frac{Max_{\text{(Pj - Gj)}}}{Max_{\text{Pj}}}$$

Max and Standard Deviation

When **Show Gaussian fit** and **Show Max & Standard Deviation** are both enabled, a vertical red line appears on the graph at the point of maximum deviation. **Max Deviation = xx.x%** and **Std. Deviation = xx.x%** are displayed in red on the graph. **Deviation** is defined at each position on the profile, relative to the fitted Gaussian curve. If the Gaussian level is $\mathbf{G_j}$, and the profile value is $\mathbf{P_j}$, then the percent deviation at that point is defined as:

$$Deviation\% = 100 * \frac{P_j - G_j}{G_j}$$

For example, if $P_j = G_j$, the deviation is 0%. If $P_j = 2*G_j$, the deviation is 100%. The deviation value can be greater than 100% and as low as -100%. **Max Deviation** is defined as the maximum value (positive or negative) of the deviation. **Std. Deviation** is calculated over the fitted region using the deviation values as calculated above.

Top-Hat Fit

When **Show Top-Hat fit** is enabled, a **TopHat** results button appears above the profile area (Fig. 3) . A red, horizontal line is superimposed over the profile; this **reference line** is used as a reference level for subsequent Top-Hat fit calculations. The **reference line** and the other Top-Hat metrics are calculated as follows:

1. Determine the 50% of peak outer edges of the profile.
2. Define the center (as opposed to centroid) of the beam as the midpoint between these two points.
3. Determine the mean level of the central 80% of this region. This level is displayed as the **reference line** and defines 100% for the purpose of subsequent Top-Hat fit calculations.
4. Let $\mathbf{S_{deviations}}$ equal the sum of the absolute value of deviations for all points.
5. Let $\mathbf{A_{reference}}$ equal the area under the reference line.
6. The **TopHat** result button is displayed as a fit % and is calculated using

$$TopHat\% = 100 * (1 - \frac{S_{deviations}}{A_{reference}})$$

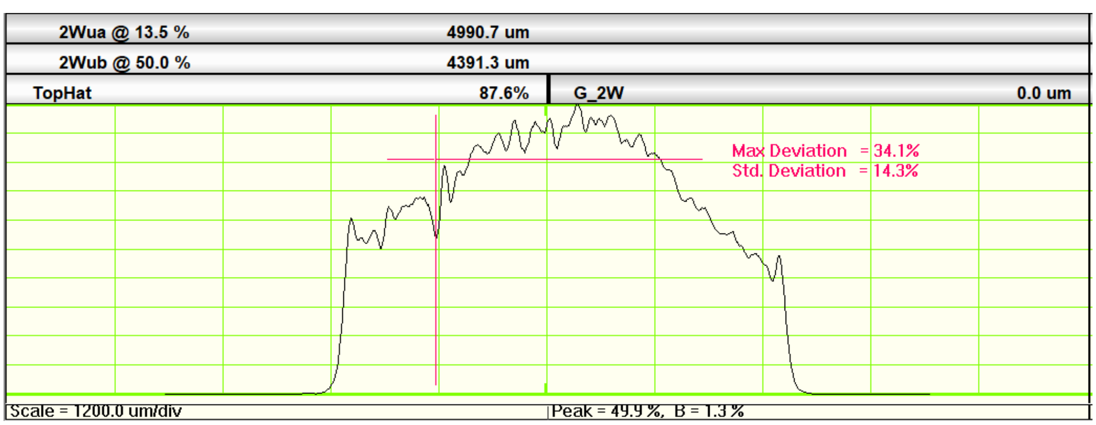

Figure 3: When **Show Top-Hat fit** is enabled, the **TopHat** result button is displayed above the profile area. The **reference line** is superimposed in the color red. The vertical red line marks the point of **Max Deviation** between the **reference line** and the profile data.

Max and Standard Deviation

When **Show Top-Hat fit** and **Show Max & Standard Deviation** are both enabled, a vertical red line appears on the graph at the point of maximum deviation. **Max Deviation = xx.x%** and **Std. Deviation = xx.x%** are displayed in red on the graph. **Deviation** is defined at each position on the profile, relative to the Top-Hat **reference line**. If the **reference line** level is $\mathbf{R_j}$, and the profile value is $\mathbf{P_j}$, then the percent deviation at that point is defined as

$$Deviation\% = 100 * \frac{P_j - R_j}{R_j}$$

For example, if $P_j = R_j$, the deviation is 0%. If $P_j = 2*R_j$, the deviation is 100%. The deviation value can be greater than 100% and as low as -100%. **Max Deviation** is defined as the maximum value (positive or negative) of the deviation. **Std. Deviation** is calculated over the fitted region using the deviation values as calculated above.

Non-uniformity

When **Show Uniformity within Clip A Diameter** is enabled, **Non-uniformity** is calculated and displayed in red in the profile area. It is calculated as follows:

1. Determine the **Clip A** edges of the profile.
2. Define **max** as the greatest value in the central 90% of this region.
3. Defome **min** as the lowest value in the central 90% of this region.
4. Calculate **Non-uniformity** as a percentage using:

$$NonUniformity\% = \frac{max - min}{max + min}$$